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MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

October/November 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 Find the exact value of $\int_1^2 \ln 3x \, dx$. Give your answer in the form $a + \ln b$, where a and b are integers. [4]

[illegible]

- 2 (a) Show that the equation $\log_4(2x+1) = 2\log_4(3x-1) - 2$ can be written as a quadratic equation in x . [3]

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- (b) Hence solve the equation $\log_4(2x+1) = 2\log_4(3x-1) - 2$. [2]

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3 (a) Express $3\sqrt{2}\sin(x+45^\circ) + \cos x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

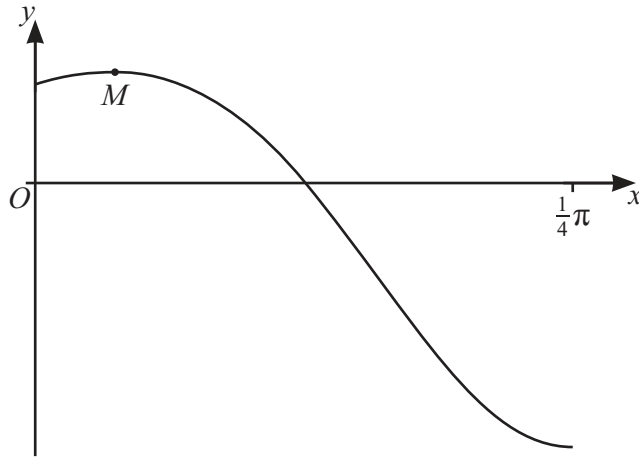
This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(b) Hence solve the equation $3\sqrt{2}\sin(3\theta+45^\circ)+\cos 3\theta=-4$ for $0^\circ < \theta < 180^\circ$.

[4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



The diagram shows the graph of $y = e^{\sin 2x} \cos 4x$ for $0 \leq x \leq \frac{1}{4}\pi$, and its maximum point M .

Find the x -coordinate of M .

[5]

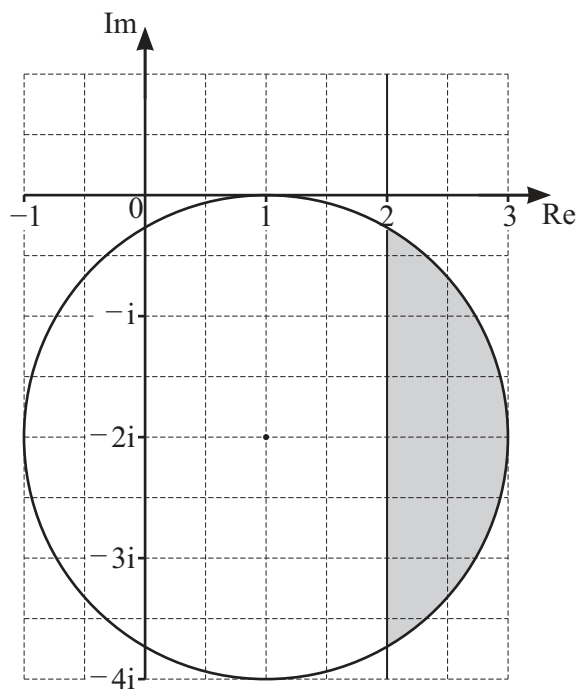
[illegible]



7

[illegible]





The shaded region on the Argand diagram shows points representing complex numbers z defined by two inequalities. The shaded region is bounded by a circle and a line parallel to the imaginary axis. The boundaries of the region are included in the shaded region.

- (a) Find two inequalities in terms of z that define the shaded region. [3]

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- (b) Calculate the least value of $\arg z$ for points in this region. [3]

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- 6** Solve the quadratic equation $(2+i)w^2 + 4w + 2-i = 0$. Give your answers in the form $x+iy$, where x and y are real. [5]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The background is white, and there are no margins or other markings present.



7 The parametric equations of a curve are

$$x = t^2 - \ln(2t + 1), \quad y = \frac{t}{2t + 1}.$$

Obtain a simplified expression for $\frac{dy}{dx}$ in terms of t .

[5]





8 The variables x and y satisfy the differential equation

$$(x^2 + 1) \frac{dy}{dx} = kxe^{2y},$$

where k is a constant. It is given that $y = 0$ when $x = 0$ and that $y = -\frac{1}{2}$ when $x = 1$.

Solve the differential equation and find the exact value of y when $x = \sqrt{3}$. [8]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

[illegible]



9 (a) By sketching a suitable pair of graphs, show that the equation

$$\sec 2x = -e^x$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(b) Show by calculation that this root lies between 0.9 and 1. [2]

[illegible]

(c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2} \cos^{-1}(-e^{-x_n})$$

converges, then it converges to the root of the equation in part (a).

[1]

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(d) Use the iterative formula given in part (c) to calculate x correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

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10 Let $f(x) = \frac{x^3 + 2x - 11}{(3+x)(2+x^2)}$.

(a) Express $f(x)$ in partial fractions.

[6]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

[illegible]

- 11 With respect to the origin O , the points A , B , C and D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}.$$

The line m passes through the points A and B .

- (a) Find a vector equation for m .

[2]

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- (b) Find the position vector of the point of intersection of m and the line passing through the points C and D .

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(c) Find the position vector of the foot of the perpendicular from C to m .

[4]

[illegible]



Additional page

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[illegible]

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